Terrain-Related Gravimetric Quantities Computed for the Next EGM

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Abstract. The development of a new Earth Gravitational Model (EGM) to degree 2160 is underway within the National Geospatial-Intelligence Agency (NGA) of the USA and its supporting group at SGT, Inc.. Among other things, this endeavor requires the compilation of a very high-resolution global topographic database, to be used consistently in the computation of all terrain-related quantities necessary for the pre-processing of gravity data and for the development and subsequent use of the new EGM. Such quantities include Residual Terrain Model (RTM) effects, analytical continuation terms (g_1), Topographic/Isostatic gravitational models, and models necessary to convert height anomalies to geoid undulations. Given the very high degree of the new EGM, all these quantities and models have to be computed at a sufficiently high resolution. Towards this goal, we have compiled a global 30″×30″ Digital Topographic Model (DTM2006.0), relying heavily on elevation information made available from the Shuttle Radar Topography Mission (SRTM). We have computed, over all land areas, RTM effects and g_1 analytical continuation terms using the DTM2006.0 30″×30″ data. We have also used 5′×5′ and 2′×2′ versions of DTM2006.0 to compute models of the Topographic/Isostatic gravitational potential complete to degree 2160. In this paper we present these results and discuss their possible use for the development of the new EGM.

Keywords. Digital Terrain Model, Earth Gravitational Model, Residual Terrain Model, Forward Modeling, Analytical Continuation

1 Introduction

The pre-processing and analysis of the detailed surface gravity data necessary to support the development of an Earth Gravitational Model (EGM) complete to harmonic degree and order 2160, depends critically on the availability of accurate topographic data, at a resolution sufficiently higher than the resolution of the area-mean gravity anomalies, which will be used eventually for the development of the EGM. In *Lemoine et al.* (1998, Section 2.1) *Factor* discusses some of the uses of such topographic data within the context of the development and the subsequent use of a high-resolution EGM. These include the computation of Residual Terrain Model (RTM) effects, the computation of analytical continuation terms (g_1), the computation of Topographic/Isostatic gravitational models that may be used to "fill-in" areas void of other data, and the computation of models necessary to convert height anomalies to geoid undulations. For these computations to be made consistently, it is necessary to compile first a high-resolution global Digital Topographic Model (DTM), whose data will support the computation of all these terrain-related quantities.

2 The DTM2006.0 Database

For EGM96 (*Lemoine et al.*, 1998), which was complete to degree and order 360, a global digital topographic database (JGP95E) at 5′×5′ resolution was considered sufficient. JGP95E was formed by merging data from 29 individual sources, and, as acknowledged by its developers, left a lot to be desired in terms of accuracy and global consistency. Since that time, thanks primarily to the Shuttle Radar Topography Mission (SRTM) (*Werner,* 2001), significant progress has been made on the topographic mapping of the Earth from space. During approximately 11 days in 2000 (February 11-22), the SRTM collected data within latitudes 60°N and 56°S, thus covering approximately 80% of the total landmass of the Earth with elevation data of high, and fairly uniform, accuracy. *Rodriguez et al.* (2005) discuss in detail the accuracy characteristics of the SRTM elevations. Comparisons with ground control points whose elevations were determined independently using kinematic GPS positioning, indicate that the 90% absolute error of the SRTM elevations ranges from ±6 to ±10 meters, depending on the geographic area (*ibid.,* Table 2.1). Additional information regarding the SRTM can be obtained from the web site of the United States' Geological Survey (USGS) (http://srtm.usgs.gov/), and from the web site of NASA's Jet Propulsion Laboratory (http://www2.jpl.nasa.gov/srtm). Unfortunately, no error estimates associated with the individual SRTM elevation values were available to us.

We compiled DTM2006.0 by overlying the SRTM data over the data of DTM2002 (*Saleh and Pavlis*, 2003). In addition to the SRTM data, DTM2006.0 contains ice elevations derived from ICESat laser altimeter data over Greenland (*Ekholm, personal communication,* 2005) and over Antarctica (*DiMarzio, personal communication,* 2005). Over Antarctica, we have also used data from the "BEDMAP" project (http://www.antarctica.ac.uk/aedc/bedmap/) to define ice and water column thickness. Over the ocean, DTM2006.0 contains essentially the same information as DTM2002, which originates in the estimates of bathymetry from altimetry data and ship depth soundings of *Smith and Sandwell* (1997). DTM2006.0 was compiled in 30″×30″ resolution (providing height and depth information only), and in 2′×2′ and 5′×5′ resolutions, where lake depth and ice thickness data are also included. DTM2006.0 is identical to DTM2002 in terms of database structure and information content.

3 Harmonic Models of Elevation-related Quantities

3.1 Topography

We define the spherical harmonic expansion of mean values of an elevation-related quantity $\overline{H}_{ij}$ as:

$$\overline{H}_{ij} = \overline{H}(\theta_i, \lambda_j) = \frac{1}{\Delta\sigma_i} \sum_{n=0}^{N} \sum_{m=-n}^{n} \overline{H}_{nm} \cdot \overline{IY}_{nm}^{ij} \quad , \tag{1}$$

with the area element:

$$\Delta\sigma_i = \Delta\lambda \int_{\theta_i}^{\theta_{i+1}} \sin\theta d\theta = \Delta\lambda \cdot \left(\cos\theta_i - \cos\theta_{i+1}\right) \quad , \tag{2}$$

and the integrated surface spherical harmonic:

$$\overline{IY}_{nm}^{ij} = \int_{\theta_i}^{\theta_{i+1}} \overline{P}_{n|m|}(\cos\theta) \sin\theta d\theta \times \int_{\lambda_j}^{\lambda_{j+1}} \begin{Bmatrix} \cos m\lambda \\ \sin|m|\lambda \end{Bmatrix} d\lambda \quad \begin{matrix} \text{if } m \geq 0 \\ \text{if } m < 0 \end{matrix} \quad . \tag{3}$$

θ_i is the co-latitude and λ_j the longitude associated with a cell on the *i*-th "row" and *j*-th "column" of a global equiangular grid. N is the maximum degree and order of the expansion, and $\overline{H}_{nm}$ are the fully normalized spherical harmonic coefficients associated with the data $\overline{H}_{ij}$. $\overline{P}_{n|m|}$ are fully-normalized Associated Legendre functions. We define the degree variance $\overline{H}_n$ at degree n to be:

$$\overline{H}_n = \sum_{m=-n}^{n} \left(\overline{H}_{nm}\right)^2 \quad . \tag{4}$$

Starting from the 2′×2′ DTM2006.0 mean elevations we computed two separate sets of $\overline{H}_{nm}$ coefficients, both complete to degree and order 2700. For the first expansion, the $\overline{H}_{ij}$ values represented both heights and depths, while for the second, $\overline{H}_{ij}$ were set to zero over all oceanic cells. In both expansions the $\overline{H}_{nm}$ coefficients were estimated using a "Type 1" block diagonal least squares adjustment (see *Pavlis* in *Lemoine et al.*, 1998, Sect. 8.2.2 for details). The use of 2′×2′ area-mean values implies a sampling Nyquist degree of 5400; hence the expansions to degree and order 2700 are only marginally affected by aliasing errors (see also *Colombo*, 1981). Furthermore, for the applications related to the EGM under development (computation of RTM effects, computation of models necessary to convert height anomalies to geoid undulations), it is sufficient to use these expansions up to degree and order 2160, or 2190 at the most. Figure 1 shows the degree variances of these two expansions. As expected, the expansion representing heights only, possesses significantly less power than the one representing both heights and depths.

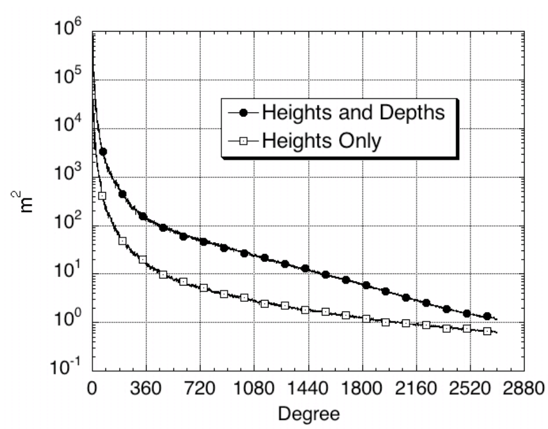

Fig. 1 Elevation-related degree variances from two expansions to *N*=2700, based on 2′×2′ DTM2006.0 data.

3.2 Topographic/Isostatic Potential

We have used the formulation described by *Pavlis and Rapp* (1990) to determine spherical harmonic coefficients of the Topographic/Isostatic (T/I) potential implied by the Airy/Heiskanen isostatic hypothesis, with a constant 30 km depth of compensation. We evaluated these coefficients up to degree and order 2160, using the DTM2006.0 data, in two ways: (a) using 5′×5′ data, and, (b) using 2′×2′ data. Figure 2 shows the gravity anomaly degree variances implied by these coefficients and by their differences.

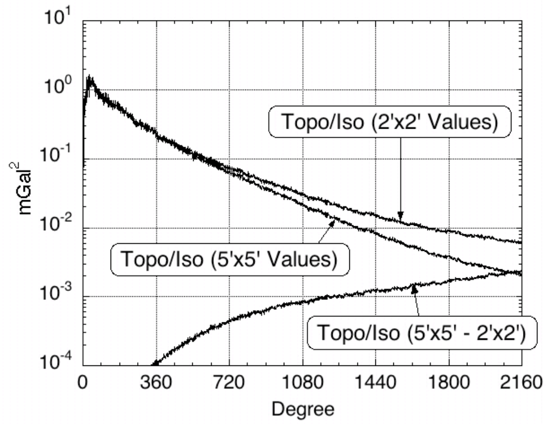

Fig. 2 Gravity anomaly degree variances implied by two estimates of the Topographic/Isostatic coefficients and by their difference (see text for details).

As expected, coefficients estimated from 2′×2′ values imply higher power than those estimated from 5′×5′ values, the difference being increasingly more significant after degree 720 or so. This suggests that in order to obtain a T/I spectrum that possesses full power (especially as this pertains to the Topographic potential), one may have to use a very high-resolution DTM (e.g., 30″×30″). Figure 3 shows the gravity anomaly spectra of the Topography only, its Isostatic compensation, and their combination (T/I), from the estimation using the 2′×2′ values.

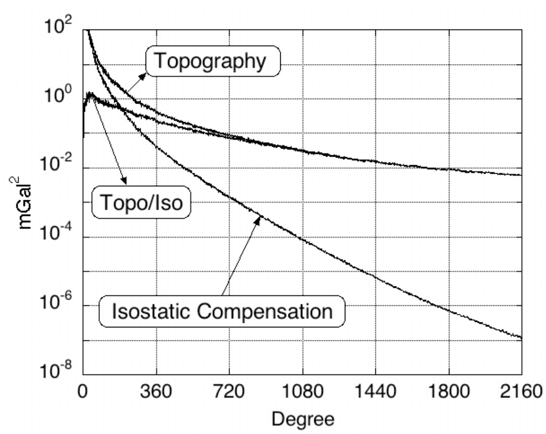

Fig. 3 Gravity anomaly degree variances of the potential of the Topography, its Isostatic compensation, and their combination (Topo/Iso).

As expected, isostatic compensation, being of regional character, has limited effect on the T/I spectrum beyond degree 720 or so.

As in the case of several global gravitational models developed previously (e.g., OSU89A/B *Rapp and Pavlis*, 1990; EGM96 *Lemoine et al.*, 1998), within the development of the new EGM, the T/I model is intended to aid the creation of synthetic gravity anomaly values. These will be used to "fill-in" areas where actual gravity data are unavailable, or their spectral content beyond some degree *n,* where $(360 \le n \le 720)$, cannot be used due to proprietary data issues. In the past, some geophysicists have criticized this practice, since it forces into the geopotential solution the isostatic hypothesis that underlies the T/I model's development. This renders the geopotential model useless for some geophysical applications, at least over the regions filled-in with the T/I gravity anomalies, and over the wavelengths implied by the degree range of the T/I coefficients used. With these considerations in mind, and in view of the implications of Figures 2 and 3, we decided to test also an alternative approach for the creation of the synthetic "fill-in" gravity anomalies that is free of any isostatic hypothesis. This approach is discussed next.

4 Forward Modeling Using RTM Gravity Anomaly Spectra

We used the 30″×30″ data of the DTM2006.0 database to compute over *all* of the Earth's landmass (including a margin extending into the ocean), a 30″×30″ grid of the gravity anomalies (Δg) implied by a Residual Terrain Model (RTM). This RTM was referenced to a topographic surface, created from the elevation harmonic coefficients described in Section 3.1, to degree and order 360. We computed the RTM Δg as described in detail by *Forsberg* (1984). We then formed 2′×2′ area-mean values of these RTM Δg, and supplemented this (primarily) land dataset with zero values for the 2′×2′ cells that are located over ocean areas (excluding the margin mentioned above). In this fashion we created a *global* 2′×2′ RTM Δg dataset. We then analyzed harmonically this dataset, and computed the ellipsoidal harmonic coefficients of the RTM Δg up to degree and order 2700. For degrees $(n < 360)$ these coefficients are small (and of no further use to us) due to the use of a reference topographic surface to degree 360. Figure 4 shows the anomaly degree variances of these RTM Δg.

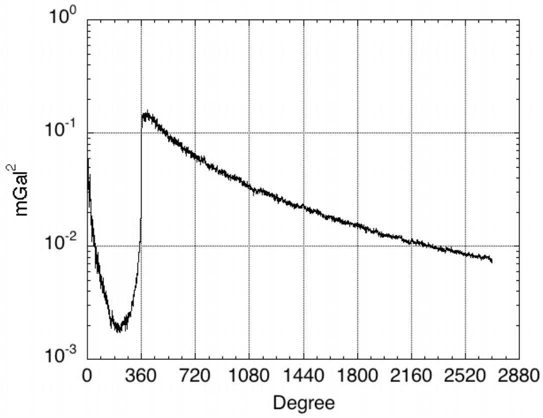

Fig. 4 Residual Terrain Model (RTM) gravity anomaly degree variances.

These ellipsoidal harmonic coefficients of the RTM Δg allowed us to synthesize "fill-in" values as follows:

(a) A global $5' \times 5'$ Δg file that includes proprietary data was created (and kept) within NGA. Over areas void of *any* gravity anomaly data, proprietary or not, (e.g., Antarctica and some areas in South America and Africa), the $5' \times 5'$ Δg were synthesized from GGM02S (*Tapley et al.,* 2005) $(n \leq 60)$, augmented with EGM96 (*Lemoine et al.,* 1998) $(61 \leq n \leq 360)$, and further augmented with the RTM Δg coefficients for $(361 \leq n \leq 2160)$.

(b) NGA personnel analyzed harmonically this dataset, and computed the ellipsoidal harmonic coefficients of these Δg values. NGA provided to us *only* the anomaly degree variances from this expansion, to degree 2160.

(c) Lower degree coefficients of the above expansion (up to some maximum degree commensurate with the minimum cell size that the use of the proprietary data is unrestricted) were then augmented with higher degree coefficients of the RTM Δg expansion. In this fashion, NGA created synthetic "cut-and-paste" model(s), all extending to degree and order 2160.

(d) Using such "cut-and-paste" model(s), synthetic "fill-in" $5' \times 5'$ Δg values were created for all the areas occupied by proprietary data (as well as for the areas void of *any* gravity anomaly values).

(e) The collection of all these "fill-in" $5' \times 5'$ Δg values, along with the unrestricted $5' \times 5'$ data, constitutes the *global* $5' \times 5'$ Δg file that NGA made available to us for further analysis.

This approach allowed NGA to provide us a global $5' \times 5'$ Δg database that does not include proprietary information.

We tested the effectiveness of this approach *globally*, by comparing the anomaly degree variances obtained from step (b), to the anomaly degree variances that we obtained from the harmonic analysis of the unrestricted $5' \times 5'$ Δg database that we received from NGA. Figure 5 shows these spectra.

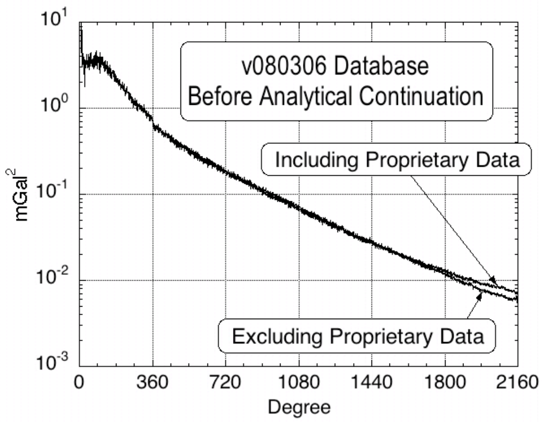

Fig. 5 Gravity anomaly degree variances implied by the analysis of two global $5' \times 5'$ gravity anomaly databases (one including and one excluding proprietary data).

As it can be seen from Figure 5, the degree variances obtained from the analysis of the unrestricted data are in excellent agreement with those obtained from the proprietary data. Only after degree ~1650 the unrestricted data analysis provides a systematically underpowered spectrum. Figure 5 indicates that the forward modeling approach using the RTM anomaly spectra, which we have devised and implemented, circumvents the proprietary data issues without degrading the gravitational solution significantly (at least in terms of the recovered power spectrum).

We also tested the effectiveness of the approach *locally*, as follows. Over areas with high quality, unrestricted $5' \times 5'$ gravity anomalies, (e.g., USA, Australia) we compared the actual data to synthetic values created from pairs of harmonic coefficient sets. Each pair contains: (a) a gravitational expansion truncated to some maximum degree Nmax (denoted by G_Nmax), and, (b) a corresponding expansion that is augmented beyond degree Nmax and up n=2160, with the coefficients implied by the RTM Δg (denoted by G+RTM_Nmax). Figure 6 shows the discrepancies between the actual $5' \times 5'$ data and the synthetic values over the USA, for Nmax=360 and Nmax=720.

It is clear from Figure 6, that augmenting the lower degree gravitational expansion with the higher degree coefficients of the RTM Δg expansion, improves significantly the agreement with the actual $5' \times 5'$ data. This, as expected, is especially true over mountainous regions like the Rocky Mountains. An obvious shortcoming of our RTM-based forward modeling approach is that it can only improve the modeling of short wavelength gravitational signals, if these signals are correlated with the topography. Table 1 summarizes the results from comparisons over the USA.

Table 1. Statistics of differences between actual 5′ gravity anomalies and synthetic values over the USA (mGal).

Model	Min.	Max.	Mean	S. Dev.
G_360	-144	150	0.2	16.3
G-540	-123	140	0.2	13.4
G_720	-137	125	0.2	11.5
G+RTM_360	-57	107	0.3	8.7
G+RTM_540	-51	84	0.3	6.5
G+RTM_720	-51	61	0.3	5.4

Table 1 shows that over the USA, augmenting the gravitational model with the RTM coefficients, reduces the standard deviation of the differences between actual and synthetic $5' \times 5'$ gravity anomalies by about a factor of two. Corresponding comparisons over Australia showed considerably less improvement obtained by augmenting the gravitational expansion with the RTM coefficients, compared to Table 1. This is because the terrain in Australia is generally less mountainous than over the USA.

5 Summary and Future Work

In preparation for a new EGM complete to degree 2160, we have compiled a new $30'' \times 30''$ global DTM (DTM2006.0). We have used its data to evaluate various terrain-related quantities, including RTM-implied Δg, Topographic/Isostatic potential coefficients, and g_1 analytical continuation terms, which may be used to analytically continue surface gravity anomalies to the ellipsoid. The computed g_1 terms represent an approximation to the *linear* gravity anomaly gradient, *assuming* linear correlation between free-air anomaly and elevation (see *Wang*, 1987). We are currently investigating if this approximation is adequate for the expansions to degree 2160, or if a better continuation procedure can be implemented based on the iterative computation of a Taylor series employing considerably higher order gradients, computed from the harmonic coefficients themselves. We have also analyzed harmonically topographic elevations, RTM-implied gravity anomalies, and the g_1 terms. We have devised and implemented successfully a forward modeling technique using the RTM anomaly spectrum, to circumvent proprietary data issues.

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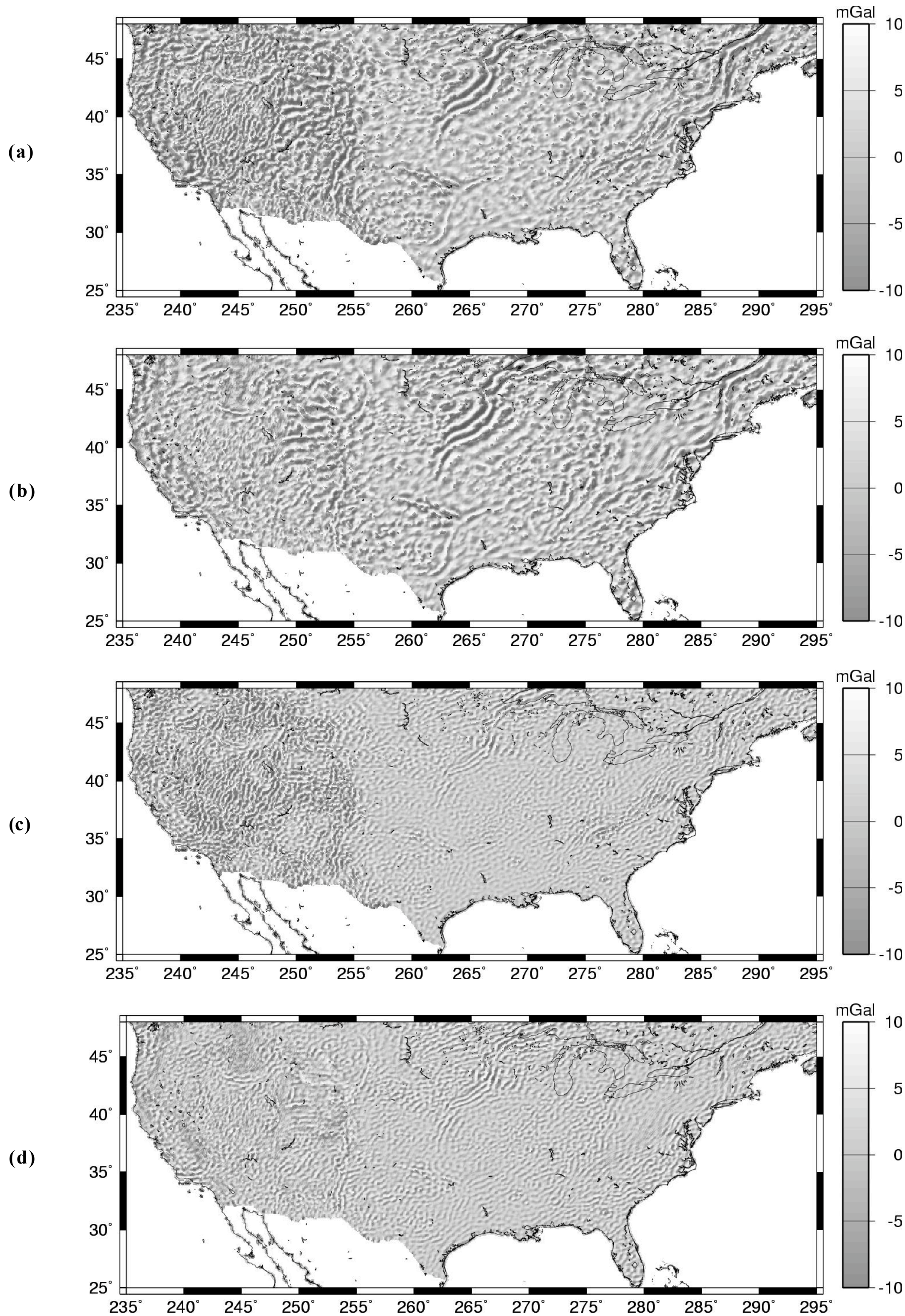

Fig. 6 Discrepancies between the 5′×5′ NGA mean gravity anomalies and synthetic values over the continental United States, with the synthetic values constructed from synthesis using: (a) gravity *($0 \leq n \leq 360$)*; (b) gravity ($0 \leq n \leq 360$) plus RTM ($361 \leq n \leq 2160$); (c) gravity ($0 \leq n \leq 720$); (d) gravity ($0 \leq n \leq 720$) plus RTM ($721 \leq n \leq 2160$).